

# Solving the Hierarchy Problem without Supersymmetry or Extra Dimensions: An Alternative Approach

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## Abstract

In this paper, we propose a possible new approach towards solving the gauge hierarchy problem without supersymmetry and without extra spacetime dimensions. This approach relies on the finiteness of string theory and the conjectured stability of certain non-supersymmetric string vacua. One crucial ingredient in this approach is the idea of “misaligned supersymmetry”, which explains how string theories may be finite even without exhibiting spacetime supersymmetry. This approach towards solving the gauge hierarchy problem is therefore complementary to recent proposals involving both large and small extra spacetime dimensions. This approach may also give a new perspective towards simultaneously solving the cosmological constant problem.

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# 1 Introduction

In recent years, there has been much discussion of new methods of solving the gauge hierarchy problem. In general, these approaches fall into several categories:

- *Weak-scale supersymmetry* [1]: This approach has the benefit of stabilizing the desert between the electroweak symmetry-breaking scale and the fundamental high-energy scales such as the GUT scale, the Planck scale, and the string scale. Weak-scale supersymmetry also has many other virtues, among them the unification of gauge couplings, a possible triggering of electroweak symmetry breaking, and the prediction of various possible dark-matter candidates. Unfortunately, the paradigm of weak-scale supersymmetry is also fraught with a number of difficulties. For example, since supersymmetry must be broken, we face the difficult question of how this breaking occurs and whether a hidden sector (and consequently a messenger sector) of some sort must be postulated in order to accomplish this. A more serious difficulty, however, is that weak-scale supersymmetry must be broken in the TeV range. Since this sets the scale for supersymmetry breaking, we cannot rely on weak-scale supersymmetry for a simultaneous solution to the cosmological constant problem.
- *Large Extra Spacetime Dimensions* [2, 3]: Over the past three years, it has become understood that large extra spacetime dimensions have the potential to lower the fundamental high-energy scales of physics, such as the string scale, the Planck scale, and the GUT scale. As such, this approach towards solving the gauge hierarchy problem eliminates (rather than stabilizes) the desert, but in certain cases this occurs at the cost of reintroducing a desert in the required compactification radii. Moreover, although weak-scale supersymmetry is no longer required for the purposes of stabilizing the hierarchy, supersymmetry of some sort (*e.g.*, in the bulk, or on distant branes) may still be required for the stability of the theory and of the brane configurations it requires. Most difficult, however, is the cosmological constant problem. Once again, this problem is not automatically solved simply by lowering the GUT, Planck, or string scales into the TeV range, and an additional mechanism still seems to be required.
- *“Warped” Spacetimes* [4]: More recently, it has been shown that even *small* extra spacetime dimensions can generate a large hierarchy between the Planck scale and the electroweak scale as a result of the warping of spacetime that occurs near a brane-like configuration of stress energy. Unlike the scenario involving large extra spacetime dimensions, this idea has the virtue of generating the hierarchy in a natural way. However, this approach also faces a number of outstanding challenges. First, it is not clear whether or how the required preconditions for this scenario can be generated from string theory. Second, it does not address (in and of itself) the cosmological constant problem; additional

mechanisms of various sorts still seem to be required [5]. Finally, supersymmetry may again be necessary in the full higher-dimensional theory in order to guarantee the fine-tunings of brane tensions that are required in this approach.

- *Conformality* [6]: Finally, there exists another approach based on the concept of “conformality”. The basic idea is that new (as yet undiscovered) states may exist in the TeV range such that, with these states included, the full theory becomes conformal (scale-invariant) and has vanishing beta-functions, even while remaining non-supersymmetric. Given this conformality, the theory essentially experiences no “running” beyond the TeV scale; all scale-sensitivity is lost, and the technical hierarchy problem is therefore solved. This approach is similar in spirit to that based on weak-scale supersymmetry: one simply replaces one symmetry (supersymmetry) with another (conformal symmetry) above the TeV scale, and both approaches (and indeed all of the above approaches) predict distinctly new physics near the TeV range. However, once again, this approach faces a number of difficulties. First, one must have a way of breaking the conformal symmetry in the TeV range; this seems to be a particularly challenging issue given the presumed scale invariance of the theory. Second, this approach does not shed any light on the cosmological constant problem; just as with weak-scale supersymmetry, one obtains a cosmological constant in the TeV range if the underlying symmetry is broken in the TeV range. Finally, it has been pointed out [7] that within this scenario, even the hierarchy problem may not be completely solved.

The purpose of this paper is to propose an alternative approach towards addressing the hierarchy and cosmological constant problems. Unlike previous approaches, we will not employ either supersymmetry or extra spacetime dimensions; instead, our primary motivation comes directly from studies of non-supersymmetric string theories and their physical spectra. We also hasten to point out that we will not be completely solving the hierarchy problem (nor for that matter the cosmological constant problem), nor will we be presenting a specific model that accomplishes all of these tasks. Rather, our purpose is simply to present an alternative philosophy towards thinking about these problems.

We also point out that for our purposes, the term “gauge hierarchy problem” will be taken to refer to the technical question of the *stability* of the electroweak scale rather than its origin. Likewise, we will define the cosmological constant problem as the problem of explaining why this constant vanishes (or at least is insensitive to heavy scales) [8].

## 2 Developing an alternative approach: Four guiding principles

Our approach ultimately rests on four guiding principles which form the core of the proposal.

- Our first guiding principle is that *the hierarchy problem and cosmological constant problem are essentially the same problem*. Indeed, both are problems of stabilizing a light scale (either the electroweak symmetry-breaking scale or the scale of the cosmological constant itself) against the effects of a heavy fundamental scale (*e.g.*,  $M_{\text{GUT}}$ ,  $M_{\text{Planck}}$ , or  $M_{\text{string}}$ ) with which it communicates quantum-mechanically. Given the commonality of these two problems, it is natural to expect that they should have a common, simultaneous solution. Unfortunately, none of the standard approaches has this feature. Although supersymmetry stabilizes the gauge hierarchy and also predicts a vanishing cosmological constant, the need to break supersymmetry in the TeV range leads to a cosmological constant which is also in the TeV range. The approach based on conformal symmetry also has a similar problem.

Indeed, on general grounds, if a symmetry (such as supersymmetry) is to protect the hierarchy but must be broken in the TeV range, then any protection it provides for the cosmological constant is also lost in the TeV range. This then leads to our second guiding principle:

- We seek a symmetry *which stabilizes the gauge hierarchy, but which does not need to be broken*. This implies that our symmetry should already be consistent with the Standard Model at low energies, which in turn implies that when unbroken, our symmetry must not intrinsically predict degenerate superpartners, conformal partners, or other light states that are not seen.

Our third guiding principle concerns the way in which we should think about energy scales. In field theory, we are encouraged to think of a linear ordering of energy scales from lowest to highest, as if arranged along a line. Moving from high energy scales to low energy scales, the heavy states “decouple”; we can then integrate them out to obtain effective theories at lower energies.

String theory, by contrast, teaches us something different: near at the fundamental string scale, we cannot necessarily distinguish heavy from light, small from large. This has often been attributed to the “quantum geometry” of string theory. The simplest example of this is the phenomenon of T-duality for closed strings: a closed string propagating in an extra spacetime dimension compactified on a circle of radius  $R$  has the same physics (*i.e.*, the same physical spectrum, the same scattering amplitudes, *etc.*) as it would have if the spacetime were compactified instead on a circle of radius  $1/R$  (in units of  $M_{\text{string}}$ ). This suggests that near the string scale, we should abandon

our usual field-theoretic notions by which we order our energy scales in a linear fashion. This observation becomes particularly relevant for phenomenology given the fact that the fundamental string scale may be anywhere between the Planck scale and the TeV scale.

Indeed, if we look within string theory to see how the miracle of T-duality arises, we find that it is achieved through a conspiracy between physics at *all energy scales simultaneously*. One cannot “integrate out” the heavy string states (*e.g.*, winding-mode states) while retaining light string states (*e.g.*, Kaluza-Klein states), since T-duality is achieved only through internal symmetries that relate these states to each other throughout the string spectrum. In other words, T-duality is intrinsically string-theoretic, and does not survive into an effective field theory which might be generated by integrating out heavy states. We are therefore led to ask:

- Can we solve the hierarchy and cosmological constant problems this way, through a conspiracy involving physics at all scales simultaneously? In such an approach, states at all energy scales should play an equal role; no states are to be “integrated out”.

What we seek, therefore, is not a “no-scales” solution (as in the conformal approach), but rather an “all-scales” solution, one in which states at all scales play an equal role simultaneously.

Finally, again taking a cue from string theory, one of the intrinsic features of string theory is the presence of an *infinite number of states in the string spectrum*. We are referring here not only to string Kaluza-Klein states, but also to string winding states (in the case of closed strings) as well as string resonances (*i.e.*, excitations of the string itself). Like the Kaluza-Klein states, the latter states also populate all mass scales simultaneously, but their numbers grow *exponentially* as functions of mass. Together, these states are what set string theory apart from field theory, giving string theory its remarkable finiteness properties.

- Can we likewise exploit an *infinite*, exponentially growing set of states at all mass scales in order to approach the hierarchy problem?

Thus, to summarize, we seek an approach involving a symmetry which has the following properties: it need not be broken at any scale (and hence must have no conflict with the Standard Model itself); all mass scales should play an equal role in conspiring to maintain this symmetry; it should involve an infinite number of states, as suggested by string theory; and it should be capable of addressing (if not solving) the hierarchy and cosmological constant problems simultaneously.

### 3 Building a toy model

The requirements discussed above clearly amount to a tall order, and it is not readily apparent how to proceed. In order to develop some intuition, let us therefore

begin by thinking about ordinary unbroken supersymmetry. How does unbroken supersymmetry manage to solve the hierarchy and cosmological constant problems?

In general, in supersymmetric theories, the quantum-mechanical sensitivities of light energy scales (such as the Higgs mass  $m_H$  and the cosmological constant  $\Lambda$ ) to heavy mass scales (*e.g.*, a cutoff  $\lambda$ ) are governed by supertraces:

$$\begin{aligned}\delta m_H^2 &\sim (\text{Str } \mathcal{M}^0) \lambda^2 + (\text{Str } \mathcal{M}^2) \log \lambda + \dots \\ \Lambda &\sim (\text{Str } \mathcal{M}^0) \lambda^4 + (\text{Str } \mathcal{M}^2) \lambda^2 + (\text{Str } \mathcal{M}^4) \log \lambda + \dots\end{aligned}\quad (3.1)$$

In these expressions, the ellipses ‘...’ denote terms which are independent of the cutoff  $\lambda$ , and the supertraces are defined as statistics-weighted sums over the spectrum of the theory:

$$\text{Str } \mathcal{M}^{2\beta} \equiv \sum_{\text{states } i} (-1)^F (M_i)^{2\beta} . \quad (3.2)$$

Thus,  $\text{Str } \mathcal{M}^0$  (which merely counts the difference between the numbers of bosonic and fermion states in the theory) governs the quadratic divergence in the one-loop Higgs mass and the quartic divergence in the one-loop cosmological constant, while  $\text{Str } \mathcal{M}^2$  governs the logarithmic divergence in the Higgs mass and the quadratic divergence in the cosmological constant, and  $\text{Str } \mathcal{M}^4$  governs the logarithmic divergence in the cosmological constant. Of course, the supertraces relevant for the Higgs mass shift are to be evaluated over only those states which couple to the Higgs; by contrast, the supertraces relevant for the cosmological constant are to be evaluated over the complete spectrum of states in the theory.

Supersymmetry works by ensuring that each of these supertraces vanishes. Indeed, unbroken supersymmetry implies

$$\text{Str } \mathcal{M}^{2\beta} = 0 \quad \text{for all } \beta \geq 0 . \quad (3.3)$$

As a result of this feature, the Higgs mass and the cosmological constant each lose their quantum-mechanical sensitivities to heavy cutoff scales  $\lambda$ , and can remain light even after quantum-mechanical effects are included. It is in this way that unbroken supersymmetry solves the hierarchy and cosmological constant problems. Indeed, this feature persists beyond one-loop, order by order. Moreover, the cosmological constant not only loses its sensitivity to heavy scales; it actually vanishes. Thus, we see that the cancellations inherent in supersymmetry are encoded as cancellations of the mass supertraces.

Of course, in the real world, supersymmetry is broken. However, even when the supersymmetry is broken, the technical gauge hierarchy problem remains solved so long as

$$\text{Str } \mathcal{M}^0 = 0 , \quad \text{Str } \mathcal{M}^2 \lesssim (\text{TeV})^2 . \quad (3.4)$$

Thus the cancellations inherent in supersymmetry continue to function in suppressing the mass supertraces, even when the supersymmetry is softly broken.

This picture works well if we are concerned only with the gauge hierarchy problem. However, this picture becomes uncomfortable if we attempt to approach the cosmological constant problem at the same time, for the supertraces given in (3.4) imply a cosmological constant which is also in the TeV range. This is too large by many orders of magnitude. From this perspective, one might say that the problem with supersymmetry is that its supertrace cancellations occur multiplet-by-multiplet, scale-by-scale. Thus, if supersymmetry is broken at one scale, *e.g.*, the TeV scale, as indicated in (3.4), then this sets the same scale for the cosmological constant problem, destroying any possible solution to both problems simultaneously. The fundamental problem here is that *the scales for the gauge hierarchy problem and the cosmological constant problem are very different*, yet the breaking of supersymmetry sets a single scale for both problems simultaneously.

As indicated earlier, this suggests that we should not attempt to break our symmetry at *any* scale. Indeed, the only reason supersymmetry must be broken at all is that superpartners are not detected in collider experiments at energies  $E \lesssim \mathcal{O}(\text{TeV})$ ; this is what selects the TeV scale in (3.4). This then leads to the fundamental question: Is there a way to preserve (3.3) through some other symmetry which need not predict superpartners, and hence which would not need to be broken at any scale? In other words, is it possible to ensure the conditions (3.3) for vanishing supertraces *without* supersymmetry?

The conditions (3.3) are very restrictive (since they must be solved for all  $\beta$  simultaneously), and for a finite number of states it is easy to see that the only solution is to arrange all bosons and fermions in the theory into exactly degenerate pairs. This essentially restores the supersymmetric configuration of the spectrum. However, the key point is that we wish to consider theories with *infinite* numbers of states. Remarkably, it turns out that for infinite numbers of states, other non-supersymmetric boson/fermion configurations leading to vanishing supertraces are possible.

Before we can proceed, however, we must first generalize the definition of supertraces given in (3.2). While the definition (3.2) is sufficient for theories containing finite numbers of states, it clearly has the potential to diverge in theories containing infinite numbers of states. We shall therefore introduce a regulated version of the supertrace:

$$\text{Str } \mathcal{M}^{2\beta} \equiv \lim_{y \rightarrow 0} \sum_{\text{states } i} (-1)^F (M_i)^{2\beta} e^{-y M_i^2} . \quad (3.5)$$

Here the quantity  $y$  acts as a regulator which damps out the contributions from heavy states with  $M_i \gg y^{-1/2}$ ; in the limit  $y \rightarrow 0$ , the contributions of all states are included in the sum. For a spectrum containing a finite number of states, this definition reduces to (3.2), while for a spectrum containing an infinite number of states, this definition will yield convergent results for all cases of interest. At this stage, the form of this regular may seem completely arbitrary. However, we shall see in Sect. 5 that this form for the regulator is motivated by string-theoretic considerations, and respects

the underlying symmetry that we will eventually be proposing as a replacement for supersymmetry or conformal invariance.

Given the generalized definition (3.5), we now return to our original question: Is it possible to satisfy the vanishing supertrace constraints (3.3) without supersymmetry? As a warm-up exercise, let us first restrict our attention to the technical gauge hierarchy problem and ask whether we can satisfy the simpler constraints

$$\text{Str } \mathcal{M}^0 = 0, \quad \text{Str } \mathcal{M}^2 = 0 \quad (3.6)$$

without supersymmetry. As we shall see later, this will turn out to be sufficient for solving both the technical hierarchy problem *as well as* the cosmological constant problem.

To do this, we shall construct our toy model by starting with a set of bosonic states, and then balancing these with additional fermionic states as needed. Let us begin therefore with a simple set of two massless bosonic states. (We choose two bosons rather than a single boson for future convenience.) For our purposes, since we are merely counting degrees of freedom, we will not distinguish the spins of our states beyond noting whether they are bosons or fermions.

A non-supersymmetric configuration of two massless bosons clearly leads to quantum-mechanical divergences that destabilize the gauge hierarchy. The standard resolution would be to introduce two new compensating fermionic states with a mass  $\mu$ . Here  $\mu$  functions as an arbitrary mass splitting. We then have

$$\text{Str } \mathcal{M}^0 = 0, \quad \text{Str } \mathcal{M}^2 = -2\mu^2. \quad (3.7)$$

The vanishing of  $\text{Str } \mathcal{M}^0$  ensures that the quartic divergences are now cancelled. Likewise, the value of  $\mu$  is flexible and might be chosen, if we wish, so as to satisfy external phenomenological criteria. Of course, the only way to satisfy (3.6) in this context is to take  $\mu = 0$ , yielding an exact boson/fermion degeneracy.

Let us now construct our toy model by considering how such a situation might be extended to one with infinite numbers of states. To do this, let us imagine duplicating the above double bosonic/fermionic system infinitely many times at ever-increasing mass scales. Specifically, we shall imagine that we have two states for each mass  $M_n = \sqrt{n}\mu$  for  $n = 0, 1, 2, \dots$ . If the level  $n$  is even (respectively odd), we will assume that our states are bosonic (respectively fermionic). This particular form for the masses  $M_n$  is motivated by string theory and chosen for later convenience. Likewise, we shall not specify a value for the overall mass scale  $\mu$  because we wish to illustrate a general mechanism; hence we shall not “integrate out” any heavy states. Using the regulated supertrace defined in (3.5), we then find

$$\begin{aligned} \text{Str } \mathcal{M}^0 &= 2 \lim_{y \rightarrow 0} \left\{ \sum_n (-1)^n e^{-yn\mu^2} \right\} \\ &= 2 \lim_{y \rightarrow 0} \left( \frac{1}{1 + e^{-y\mu}} \right) \\ &= 1. \end{aligned} \quad (3.8)$$



Note that this formal result  $\text{Str } \mathcal{M}^0 = 1$  reflects the expected averaging between the values  $+2$  and  $0$  taken by  $\text{Str } \mathcal{M}^0$  as the summation is performed over the spectrum. Likewise, we can also calculate  $\text{Str } \mathcal{M}^2$  for our example:

$$\begin{aligned}
\text{Str } \mathcal{M}^2 &= 2 \lim_{y \rightarrow 0} \left\{ \sum_n (-1)^n n \mu^2 e^{-yn\mu^2} \right\} \\
&= 2 \lim_{y \rightarrow 0} \frac{d}{dy} \left\{ \sum_n (-1)^n e^{-yn\mu^2} \right\} \\
&= 2 \lim_{y \rightarrow 0} \frac{d}{dy} \left( \frac{1}{1 + e^{-y\mu^2}} \right) \\
&= \frac{1}{2} \mu^2 .
\end{aligned} \tag{3.9}$$

Once again, the result is *finite* even though infinitely many states with infinite masses are present.

Given these results, it is quite easy to see how we might ensure that both of our supertraces vanish. Note from (3.8) and (3.9) that this infinite system of states has exactly the same supertraces as we would obtain from a single bosonic state of mass  $\mu/\sqrt{2}$ . Thus, all we need to do is introduce a single extra *fermionic* state with mass  $\mu/\sqrt{2}$  (which we would associate with a level  $n = 1/2$ ). This combined configuration — two repeating sets of pairs of bosons/fermions at integer values of  $n$ , plus a single extra fermionic state at  $n = 1/2$  — has vanishing values for both  $\text{Str } \mathcal{M}^0$  and  $\text{Str } \mathcal{M}^2$  simultaneously.

Of course, this is nothing but a simple toy model, and we do not expect this sort of configuration to be “natural” or to arise in any well-motivated physical theory. But it does illustrate several key features:

- First, we see that *cancellation of divergences does not require a strict pairing of bosonic and fermionic states if there are infinite numbers of each*. In the above example, the presence of the single extra fermionic state destroys our naïve attempt at a pairing, yet is absolutely necessary in order to produce vanishing supertraces.
- Second, this result holds for all values of  $\mu$ ; no mass scale is preferred or selected by this procedure.
- Third, if we imagine that  $\mu$  is large (*e.g.*, if  $\mu \approx M_{\text{Planck}} \sim 10^{18}$  GeV), then the only states that might be experimentally detected are those with masses  $M_i \ll \mu$ . In our toy model, these would just be the two single massless bosonic states with which we started — a very non-supersymmetric light spectrum. However, even though we would not detect any massless or light “superpartners” for these massless bosons, their divergences will nevertheless be cancelled by an infinite “cloud” of very heavy, experimentally undetectable particles with masses  $M_n =$

$\sqrt{n}\mu$ , plus a fermion with mass  $\mu/\sqrt{2}$ . Note that a cancellation of this form requires an *infinite* number of states in this theory. The actual value of the scale  $\mu$  is irrelevant.

- Finally, we see that for the purposes of such calculations, we cannot integrate out our “heavy” states, even as  $\mu \rightarrow \infty$ . Instead, regardless of the value of  $\mu$ , keeping track of *all* states in the theory is necessary in order to analyze the full divergence structure of the low-energy states. In other words, although a *finite* number of heavy states can be expected to decouple as their masses become large, an *infinite* number of states will not.

Despite these features, the above toy model is quite unnatural for a number of reasons. Perhaps its most unnatural feature is the *ad hoc* introduction of the extra fermionic state to cancel the supertraces. By its very nature, this fermionic state is not naturally part of our unified tower of states. Another unnatural feature is our restriction to having only two states for each mass  $M_n$ . Once again taking a cue from string theory, we expect that as the mass increases, the number of states should also increase, for there are increasing numbers of ways in which one can excite the fundamental string in order to produce increasingly massive states.

Finally, another unnatural feature of our toy model is the fact that we “almost” have an exact pairing of states — indeed, except for the solitary extra fermionic state at mass  $\mu/\sqrt{2}$ , all of the other states can easily be paired and have splittings that scale with  $\mu$ . This situation is very reminiscent of configurations with broken supersymmetry. However, we are seeking scenarios in which there is no remnant of multiplet-by-multiplet supersymmetry, approximate or otherwise.

Fortunately, it is not difficult to construct boson/fermion configurations which overcome all of these difficulties. For example, let us again consider a distribution of states with masses  $M_n = \sqrt{n}\mu$ , bosonic if  $n$  is even and fermionic if  $n$  is odd. Let us further assume that the number of such states of each mass  $M_n$  is given by some number  $g_n$ , with positive (negative) values of  $g_n$  signifying bosons (fermions). Then solutions such as

$$g_n = \begin{cases} (-1)^n n^{2k} & \text{for any } k \geq 1, k \in \mathbb{Z} \\ (-1)^n (n^5 - n) \\ (-1)^n (n^5 + 2n^3) \end{cases} \quad (3.10)$$

all have the property that they satisfy (3.3). Indeed, not only do  $\text{Str } \mathcal{M}^0$  and  $\text{Str } \mathcal{M}^2$  cancel identically, but all higher mass supertraces cancel as well — all without any boson/fermion degeneracies! For example, the case with  $g_n = (-1)^n (n^5 + 2n^3)$  corresponds to an explicit configuration of low-lying bosonic and fermionic states consisting of three fermions with mass  $\mu$ ; 48 bosons with mass  $\sqrt{2}\mu$ ; 297 fermions with mass  $\sqrt{3}\mu$ ; 1152 bosons with mass  $2\mu$ ; 3375 fermions with mass  $\sqrt{5}\mu$ ; and so forth. There is no apparent way to form a state-by-state pairing between bosons and fermions, yet this configuration of bosons and fermions manages to have vanishing supertraces

of arbitrarily high order. In particular, there is no supermultiplet structure, and no extraneous counterbalancing states are required. As in all cases, however, the crucial ingredient in the success of this scenario is the existence of an infinite number of states. Indeed, it is only because of the subtle interplay between states *at all mass scales* that the supertraces are cancelled and the dependence on the scale  $\mu$  is eliminated.

However, even these solutions are not ideal. One glaring problem, for example, is the fact that no massless states arise in these solutions; all of these solutions have  $g_0 = 0$ . In the cases where  $\mu$  is large, we are therefore left with no light states. Of course, we always have the freedom to superimpose onto this configuration any other (supersymmetric) configuration with equal numbers of bosons and fermions at any mass. This does not disturb the vanishing supertrace relations that we have already managed to satisfy. However, even though this would introduce massless states into the picture, our massless world would still be required to be supersymmetric.

Another (related) problem is the fact that the degeneracies of states in these solutions grow only polynomially in  $n$ . If we wish to take the suggestions of string theory seriously, then we should seek solutions for  $g_n$  which grow *exponentially* according to the asymptotic formula

$$|g_n| \sim A n^{-B} e^{C\sqrt{n}} \quad \text{as } n \rightarrow \infty \quad (3.11)$$

where  $A$ ,  $B$ , and  $C$  are all positive constants. This is the expected asymptotic behavior for the number of ways of producing a state consisting of  $n$  total excitations using only integer excitation modes.

It turns out to be much more difficult to construct non-trivial solutions that also satisfy these two additional criteria. However, using techniques from string theory, solutions can be found. In this section, we shall simply present what we shall refer to as a “magic” solution. In Sect. 5, we shall discuss in detail how such solutions may be generated and interpreted.

Our “magic” solution consists of the following low-lying degeneracies  $g_n$ :

$n$	$g_n$	$n$	$g_n$
0	+36	6	−29, 010, 432
1	+1, 024	7	−29, 774, 848
2	−19, 712	8	+529, 050, 944
3	−76, 800	9	+410, 305, 536
4	+1, 051, 136	10	−7, 301, 403, 648
5	+1, 806, 336	11	−4, 414, 798, 848

Thus, unlike the previous cases, we see that this solution has bosons at levels  $n \in \{0, 1\} \pmod{4}$  and fermions at levels  $n \in \{2, 3\} \pmod{4}$ . Of course, in order to fully specify this solution, we need to quote more than a finite set of numbers  $g_n$ ; we need

to specify a procedure by which *all* values of  $g_n$  can be generated. This procedure will be given explicitly in Sect. 5.

This “magic” solution has a number of appealing properties. First, it is explicitly non-supersymmetric; there exists no manifest way of pairing states at any mass level in order to construct supersymmetric multiplets. Moreover, since  $g_0 \neq 0$ , we see that the lightest states are themselves non-supersymmetric; thus even as  $\mu \rightarrow \infty$  we retain a non-supersymmetric spectrum of light states. Furthermore, these degeneracies  $g_n$  exhibit\* the desired exponential growth anticipated in (3.11).

But most importantly, our “magic” solution has two critical properties. First, both of the supertraces relevant for the gauge hierarchy problem vanish identically:

$$\text{Str } \mathcal{M}^0 = \text{Str } \mathcal{M}^2 = 0 . \quad (3.12)$$

These supertrace cancellations occur as a result of a non-trivial conspiracy across the infinite particle spectrum in this theory. Thus, because of these cancellations, the gauge hierarchy ceases to depend on the splitting scale  $\mu$  or any other high fundamental scale.

Second, it turns out that the one-loop cosmological constant corresponding to this solution actually vanishes. Recall that for an arbitrary set of degeneracies  $g_n$ , the one-loop cosmological constant (or equivalently the one-loop zero-point vacuum energy) is given by

$$\Lambda = \frac{1}{2} \sum_n g_n \int \frac{d^4 p}{(2\pi)^4} \log(p^2 + M_n^2) . \quad (3.13)$$

The dependence on the degeneracies  $g_n$  is shown explicitly in this equation. However, as we shall see in Sect. 5, for this particular choice of  $g_n$  it is possible to extract a finite result for  $\Lambda$  in a self-consistent way, and moreover, this finite result actually vanishes. Thus, for the particular choice of  $g_n$  in this “magic” solution, we have not only vanishing supertraces but also  $\Lambda = 0$ . Note that this has been achieved *without* supersymmetry and without any explicit boson/fermion pairings. Moreover, this holds for all values of the splitting parameter  $\mu$ .

What are the phenomenological implications of such a scenario? One possibility is to take  $\mu \approx M_{\text{Planck}} \sim 10^{18}$  GeV and to identify the “massless” states as those of the Standard Model. Of course, the Standard Model does not consist merely of 36 bosonic degrees of freedom. However, given one solution with the above properties, it is trivial to generate others. For example, all degeneracies  $g_n$  may be subjected to an arbitrary common rescaling, since this does not affect the vanishing of the supertraces. This implies, in particular, that we can also reverse the signs of all of the degeneracies, thus exchanging bosons and fermions without affecting the final result. Moreover, at each mass level, we are free to add an arbitrary *equal* number of bosons and fermions,

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\* More precisely, each separate sequence of integers  $|g_n|$  with a different value of  $n \pmod{4}$  obeys (3.11) individually. This is sufficient for our purposes, and will be discussed further in Sect. 4.

since  $g_n$  represents only the *net* number of bosons minus fermions at the  $n^{\text{th}}$  mass level. Given these facts, it is always possible to accommodate the Standard Model states within the lightest states in such a configuration.

We therefore see that it is possible to think of the Standard Model states as being only the lightest states in a theory containing an infinite number of states. For sufficiently large mass splittings  $\mu$ , no other light states need appear, yet by properly summing over the infinite spectrum of such a theory, one finds that all supertraces cancel. Our low-energy theory consequently loses its quantum-mechanical dependence on heavy mass scales, and the *technical* hierarchy “problem” is reduced to a fictitious problem that appears only because we have integrated out an *infinite* tower of states. As we have seen, the cosmological constant problem may also be addressed simultaneously in this approach.

Clearly, this scenario is radically different from the usual scenario involving weak-scale supersymmetry. Rather than embellish the Standard Model particles with weak-scale superpartners, we are instead embellishing them with a “cloud” of infinitely many heavy states which perform essentially the same function, namely that of regulating and/or cancelling divergences. Thus, we need not impose supersymmetry and then worry about how to break it — instead, we never have it at all. Moreover, in this scenario, the new particles are not necessarily at the weak scale; the precise scale depends on the (arbitrary) value of  $\mu$ . However, because the configuration of these heavy particles has been very carefully chosen, a conspiracy between physics at all energy scales occurs, and the quantum-mechanical dependence of the low-energy theory on heavy mass scales is eliminated.

## 4 Connection to string theory

In some sense, what we have so far is merely a mathematical result concerning a new way to cancel supertraces. But the deeper question remains: why should one believe that this is relevant for the real world?

Fortunately, the best motivation for this approach is that *this is what string theory actually does*. Recall that string theory is a finite theory: the finiteness of string theory rests not on supersymmetry, but rather on the fact that strings are extended objects. It is this extended nature of the string which provides an automatic “regulator” for the ultraviolet (short-distance) divergences that normally plague quantum field theories. If the string model in question happens to exhibit spacetime supersymmetry, then this finiteness manifests itself in the usual manner, via pairwise cancellations throughout the string spectrum. However, even if the string is non-supersymmetric, this finiteness must be maintained.\* In such cases, *the mechanism we have presented in the previous section is the method by which string theory*

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\*We are here ignoring a number of subtle issues concerning the stability of non-supersymmetric strings. We will discuss these issues more carefully below.

*manages to adjust its spectrum to maintain finiteness, even without supersymmetry.* Rather than rely on the level-by-level cancellations inherent in supersymmetry, the cancellations leading to finiteness in this scenario instead occur non-trivially across the entire string spectrum in the manner just described. In fact, this sort of cancellation has been proven [9, 10] to be a general property of all closed, perturbative, tachyon-free non-supersymmetric strings.

As an example, let us consider what is perhaps the most famous perturbative non-supersymmetric string theory: this is the ten-dimensional tachyon-free heterotic string model with gauge group  $SO(16) \times SO(16)$ , originally constructed in Ref. [11]. The spectrum of this model has alternating, exponentially increasing bosonic and fermionic surpluses:

$$(g_0; g_1; g_2; \dots) = (-2, 112; 147, 456; -4, 713, 984; \dots) . \quad (4.1)$$

Moreover, as we shall discuss in Sect. 5, one finds that the first *four* mass supertraces all vanish in this model [10]:

$$\text{Str } \mathcal{M}^0 = \text{Str } \mathcal{M}^2 = \text{Str } \mathcal{M}^4 = \text{Str } \mathcal{M}^6 = 0 . \quad (4.2)$$

As shown in Refs. [9, 10], the underlying symmetry which assures that these cancellations take place is nothing but modular invariance. This is hardly surprising, since modular invariance has its roots in the extended nature of the string, and serves as a powerful constraint on the string spectrum. Thus, modular invariance (and its multiloop generalizations) is the symmetry that we are proposing to replace supersymmetry or conformal symmetry as our solution to the hierarchy problem. Indeed, modular invariance is a general property of the perturbative moduli space of all closed string theories (and the bulk sector of Type I string theories). However, unlike supersymmetry or conformal symmetry, *there is no need to break modular invariance for phenomenological purposes*. Thus, our symmetry and the cancellations it induces can remain intact even at low energies.

This also answers another question. *A priori*, the cancellation outlined in the previous section might have seemed to be a highly fragile one, since a small shift in the mass of a single state (*e.g.*, due to a radiative correction) would seem to destroy the delicate supertrace cancellations we have managed to achieve. However, these cancellations are ultimately robust because they are the manifestation of deeper symmetry, in this case modular invariance. Thus, if one state in the string spectrum is shifted due to some dynamical effect calculated within the framework of the full string theory, the rest of the string spectrum automatically compensates in a modular-invariant way. Thus, the supertrace cancellations are ultimately preserved and protected by modular invariance.

It is easy to understand explicitly how finiteness is achieved in such alternating boson/fermion scenarios. For this purpose, let us consider a typical boson/fermion configuration, as sketched in Fig. 1. As we scan through higher and higher mass

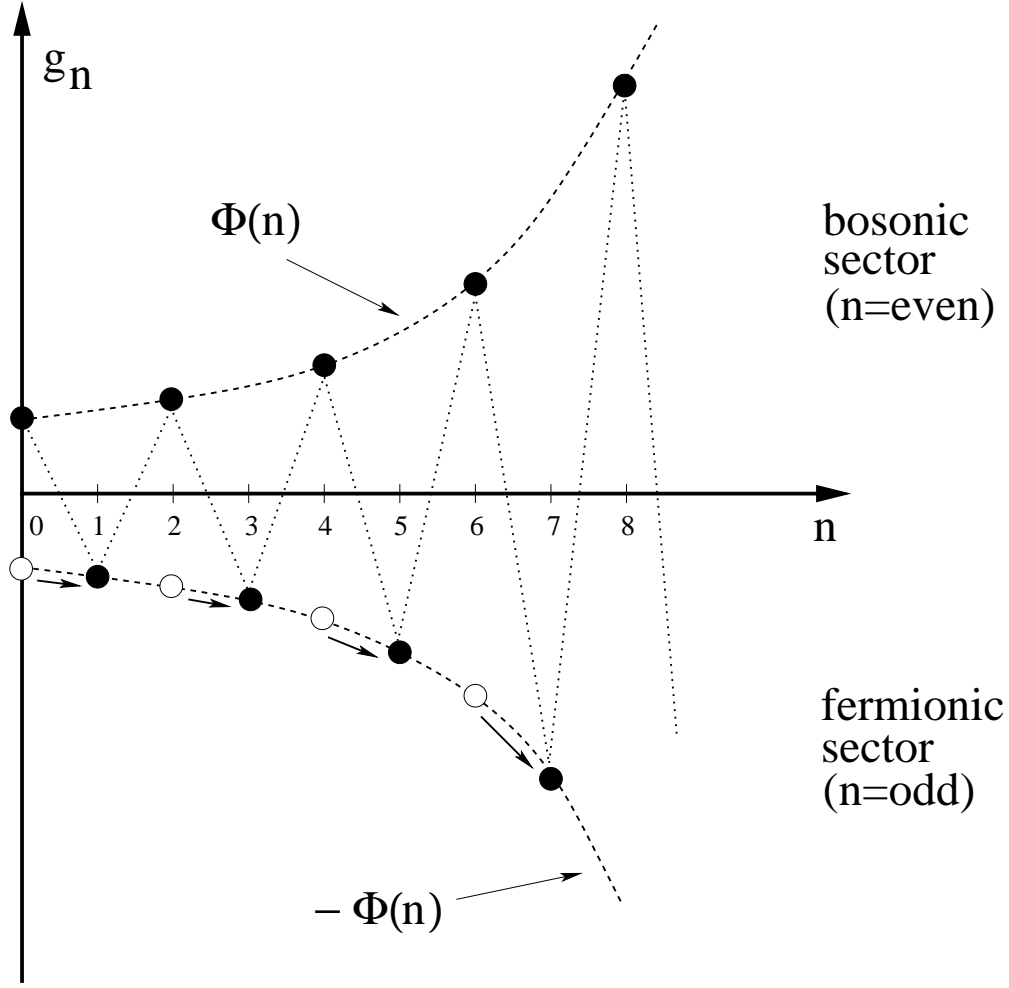


Figure 1: A sketch of a typical boson/fermion configuration. In this sketch, we have assumed a bosonic (fermionic) sector at even (odd) mass levels  $n$ , and plotted a typical configuration of degeneracies  $g_n$  (black circles) versus  $n$ . The dashed lines connect these points in order of increasing mass  $M_n = \sqrt{n}\mu$ , and illustrate the regular bosonic/fermionic oscillations inherent in “misaligned supersymmetry”. The cancellation of the supertraces arises as a result of the *cancellation of the functional forms*  $\Phi(n)$  that separately govern the behavior of  $g_n$  as a function of  $n$  in each sector. Ordinary supersymmetry emerges as a special case when the bosonic and fermionic sectors have values of  $n$  that coincide (no misalignment). We illustrate this for even values of  $n$ , with degeneracies (black circles) in the bosonic sector cancelling pairwise against degeneracies (white circles) in the fermionic sector. However, as the fermionic sector is shifted (“misaligned”) by  $n \rightarrow n + \Delta n$  relative to the bosonic sector, the white circles slide along the functional form  $-\Phi(n)$  to their new locations  $-\Phi(n + \Delta n)$  at which pairwise cancellations of states are no longer possible. Although supersymmetry is broken, finiteness is nevertheless maintained due to the cancellation of the functional forms, and the mass supertraces continue to vanish.

levels, the spectrum oscillates between exponentially growing bosonic and fermionic surpluses. In the sketch in Fig. 1, for example, the sectors with bosonic surpluses occur at mass levels where  $n$  is even, while the sectors with fermionic surpluses occur when  $n$  is odd. Clearly, in such a configuration, there is no pairwise cancellation of states. Nevertheless, in each sector separately, there exists a unique way [12, 9] of determining a smooth function  $\Phi(n)$  which describes the exponentially growing degeneracies  $g_n$  at the appropriate values of  $n$ . For large  $n$ , these functions typically take the form

$$\Phi(n) = \sum_{\ell=1}^{\infty} \sum_i f(n + n_i) \exp\left(\frac{c_i}{\ell} \sqrt{n + n_i}\right) \quad (4.3)$$

where  $\{c_1, c_2, c_3, \dots\}$  are a set of positive real numbers; where  $0 \leq n_i < 1$  for all  $i$ ; and where  $f(x)$  is a function which grows at most polynomially in  $x$ . Thus,  $\Phi(n)$  takes the form of an infinite series of leading and subleading exponentials, with the leading term, as expected, taking the Hagedorn form (3.11) for large  $n$ . Given this, the cancellation that preserves finiteness in such configurations is not a cancellation of the degeneracies  $g_n$ , but rather a cancellation of the functional forms  $\Phi(n)$  between the different sectors of the theory [9]. Specifically, if  $\Phi_b(n)$  represents the functional form describing the degeneracies  $g_n$  in the bosonic sector and  $\Phi_f(n)$  represents the analogous functional form for  $|g_n|$  in the fermionic sector, then

$$\frac{\Phi_b(n) - \Phi_f(n)}{\Phi_b(n) + \Phi_f(n)} \rightarrow 0 \quad \text{as } n \rightarrow \infty. \quad (4.4)$$

In other words, the leading (and often also the highest subleading) exponential terms in (4.3) necessarily cancel between bosonic and fermionic sectors. In fact, for *any* non-supersymmetric tachyon-free closed string model with multiple bosonic and fermionic sectors, it has been shown [9] that

$$\frac{\sum_i (-1)^{F_i} \Phi_i(n)}{\sum_i \Phi_i(n)} \rightarrow 0 \quad \text{as } n \rightarrow \infty \quad (4.5)$$

where  $F_i$  is the fermion number for the  $i^{\text{th}}$  sector. This is thus a completely general property. Moreover, it has been conjectured [9] that above cancellations even take the stronger form

$$\sum_i (-1)^{F_i} \Phi_i(n) = 0. \quad (4.6)$$

This would correspond to a complete cancellation of *all* terms within (4.3). We will discuss these results in more detail below.

In some sense, this mechanism is a generalization of the level-by-level pairing of states that occurs in ordinary unbroken supersymmetry. To see this, let us return to Fig. 1 and imagine that the bosonic and fermionic sectors were exactly aligned, so that both had mass levels only for even values of  $n$ . In this case, the degeneracies  $g_n$  for the bosonic sector would be exactly aligned with those for the fermionic sector, and the



cancellation of functional forms in (4.6) would imply an exact pairwise cancellation of states. Indeed, this is what happens for a supersymmetric string model. However, as we break supersymmetry and shift the mass levels of the fermionic sector relative to those in the bosonic sector by an amount  $\Delta n$ , the degeneracies  $g_n$  in the fermionic sector shift according to<sup>†</sup>

$$g_n = -\Phi_f(n) \quad \longrightarrow \quad g'_n = -\Phi_f(n + \Delta n) . \quad (4.7)$$

This shift is illustrated in Fig. 1. In other words, the fermionic states redistribute themselves across the entire infinite string spectrum in such a way that although the level-by-level pairing of states is destroyed, the cancellation of the functional forms is still preserved. This can therefore be called a “misaligned supersymmetry” [9]. Thus, “misaligned supersymmetry” and its associated cancellations is the underlying mechanism by which modular invariance is maintained in the string spectrum, even when supersymmetry is no longer present. In fact, as shown in Ref. [9], this is the most general way by which supersymmetry can be broken in string theory while simultaneously maintaining modular invariance (a critical symmetry for the perturbative consistency of closed strings) and avoiding the appearance of tachyonic string states.

Note that this mechanism goes beyond a mere uniform shifting of the masses of certain states. While some states become heavier because of the misalignment, other states (typically winding-mode states) rearrange themselves and become lighter such that the net values of  $g_n$  continue to fall along the same functional form  $\Phi_f(n)$ . Such a rearrangement is possible only in a theory with an *infinite* number of states.

Of course, Fig. 1 is only a sketch of an idealized situation. In actual string models, these cancellations can be far more complicated. Let us consider two examples. Our first example is a four-dimensional, non-supersymmetric, tachyon-free perturbative heterotic string model with gauge group  $SU_6 \times (SU_4)^3$ . (The full gauge group of this model is  $SU_6 \times (SU_4)^3 \times (SU_2)^9 \times U_1$ .) This model is constructed via the heterotic free-fermionic construction [13]. For this string model, we can plot the degeneracies  $g_n$  as a function of  $n$ . We then obtain the result shown in the top row of Fig. 2. Note that this model contains four sectors distributed along quarter-integer values of  $n$ ; those with  $n \in \{0, 1/4\} \pmod{1}$  have bosonic surpluses, while those with  $n \in \{1/2, 3/4\} \pmod{1}$  have fermionic surpluses. Labelling these sectors as  $\{A, B, C, D\}$  respectively, we see that there are four distinct functions  $\Phi_{A,B,C,D}(n)$  which describe the degeneracies in these sectors respectively. As indicated in Fig. 2 and as expected from (3.11), these functions are essentially linear on a log plot of  $g_n$  versus  $\sqrt{n}$ . Indeed, as evident from Fig. 2,  $\Phi_{B,D}$  have a stronger leading exponential behavior than  $\Phi_{A,C}$ . Nevertheless, it follows that

$$\frac{\Phi_A(n) + \Phi_B(n) - \Phi_C(n) - \Phi_D(n)}{\Phi_A(n) + \Phi_B(n) + \Phi_C(n) + \Phi_D(n)} \rightarrow 0 \quad \text{as } n \rightarrow \infty , \quad (4.8)$$

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<sup>†</sup> Strictly speaking, as supersymmetry is broken, the relevant values of  $n$  shift but the functional forms  $\Phi(n)$  also change in such a way as to ensure that the corresponding degeneracies  $g_n$  are always integers.

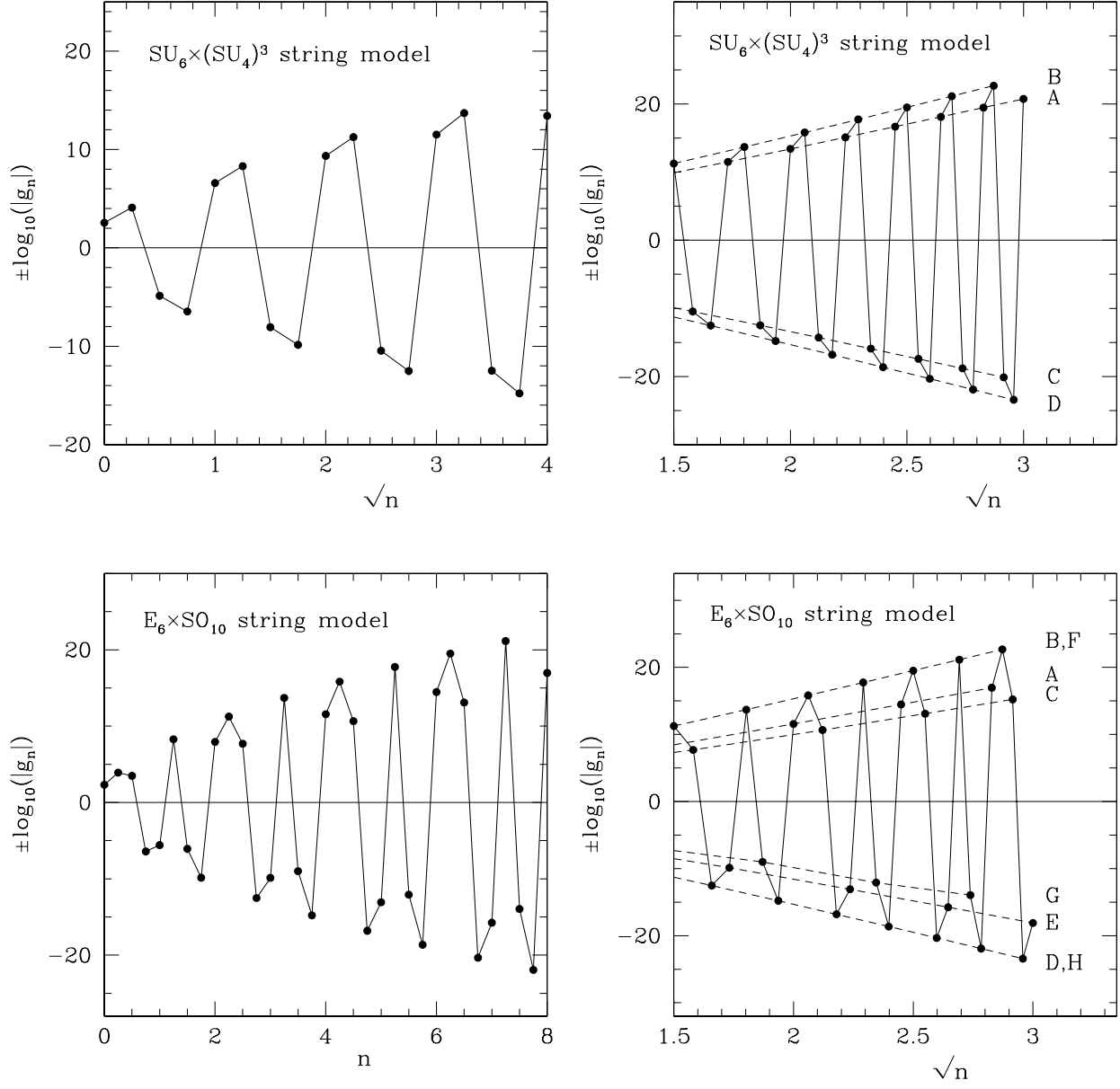


Figure 2: Degeneracies  $g_n$  as functions of  $n$  in four-dimensional non-supersymmetric tachyon-free heterotic string models with gauge groups  $SU_6 \times (SU_4)^3$  (top row) and  $E_6 \times SO_{10}$  (bottom row). In these figures we plot  $\pm \log_{10}(|g_n|)$  where the sign chosen is the sign of  $g_n$ . In all cases, modular invariance causes cancellations to occur, leading to a misaligned supersymmetry and preserving finiteness.

with the conjectured stronger cancellation

$$\Phi_A(n) + \Phi_B(n) - \Phi_C(n) - \Phi_D(n) = 0 . \quad (4.9)$$

This implies that the leading exponential behavior between  $\Phi_B$  and  $\Phi_D$  cancels exactly; that the remaining subleading exponential behavior from  $\Phi_{B,D}$  cancels against the leading exponential behavior from  $\Phi_{A,C}$ ; and so forth.

As a more complicated example, let us consider a different string model with gauge symmetry  $E_6 \times SO_{10} \times (SU_4)^3 \times (U_1)^2$ . The degeneracies for this model are plotted in the lower row of Fig. 2. Unlike the previous model, this model has *eight* distinct sectors with values of  $n \in \{0, 1/4, 1/2, 3/4, 1, 5/4, 3/2, 7/4\} \pmod{2}$ . It is immediately apparent that the oscillation pattern in this model is much more complex than it is in the previous model. Nevertheless, labelling these sectors  $\{A, \dots, H\}$  respectively, we still find the net cancellation

$$\frac{\Phi_A + \Phi_B + \Phi_C - \Phi_D - \Phi_E + \Phi_F - \Phi_G - \Phi_H}{\Phi_A + \Phi_B + \Phi_C + \Phi_D + \Phi_E + \Phi_F + \Phi_G + \Phi_H} \rightarrow 0 \quad \text{as } n \rightarrow \infty . \quad (4.10)$$

Indeed, regardless of the particular model or its method of construction, modular invariance causes cancellations to occur, leading to a misaligned supersymmetry and preserving finiteness. Moreover, in each case, it is the presence of an infinite number of states which permits such cancellations to occur.

## 5 Technical details

We shall now provide some technical background behind the results which have been presented thus far. This section is self-contained, and the reader uninterested in these details can proceed directly to the Conclusions. The purpose of this section is to provide a summary of those results in the literature which provide further details behind the claims above.

Our goal is to understand the emergence of modular invariance and its relation to misaligned supersymmetry, the mass supertraces, and the cosmological constant. Let us therefore begin by considering the field-theoretic one-loop cosmological constant. In  $D$  dimensions, this is given by

$$\Lambda = \frac{1}{2} \sum_n (-1)^{F_n} g_n \int \frac{d^D p}{(2\pi)^D} \log(p^2 + M_n^2) . \quad (5.1)$$

For  $D = 4$ , this reduces to the expression given in (3.13). Clearly, this field-theoretic expression has a divergence as  $p^2 \rightarrow \infty$ . In order to understand how the presence of an infinite number of string states enables us to eliminate this divergence, let us follow the standard prescription by rewriting this expression in terms of a Schwinger proper time  $t$  by using the identity

$$\log x = \int_1^x \frac{dy}{y} = \int_1^x dy \int_0^\infty dt e^{-yt} = - \int_0^\infty \frac{dt}{t} e^{-xt} + \dots \quad (5.2)$$

where we have dropped an  $x$ -independent term. Since we are interested in the divergence behavior as  $x \rightarrow \infty$ , it is legitimate to drop this term. We thus obtain

$$\Lambda = -\frac{1}{2} \sum_n (-1)^{F_n} g_n \int \frac{d^D p}{(2\pi)^D} \int_0^\infty \frac{dt}{t} e^{-(p^2 + M_n^2)t} , \quad (5.3)$$

and the ultraviolet divergence as  $p^2 \rightarrow \infty$  now appears as a divergence as  $t \rightarrow 0$ . Performing the momentum integrations then yields

$$\Lambda = -\frac{1}{2} \frac{1}{(4\pi)^{D/2}} \sum_n (-1)^{F_n} g_n \int_0^\infty \frac{dt}{t^{1+D/2}} e^{-M_n^2 t} . \quad (5.4)$$

In order to show how modular invariance can arise, we now shift our notation slightly by making the following substitutions. First, we define the *dimensionless* real parameter  $\tau_2$  as

$$\tau_2 = \frac{1}{4\pi} \mu^2 t \quad (5.5)$$

where  $\mu$  is an (as yet) unspecified mass scale. Second, we introduce an additional dimensionless real variable  $\tau_1$  by inserting

$$1 = \int_{-1/2}^{1/2} d\tau_1 \quad (5.6)$$

into our expressions. We then combine our two new parameters to form the complex variable

$$\tau \equiv \tau_1 + i\tau_2 , \quad (5.7)$$

thereby enabling us to rewrite our expression for  $\Lambda$  in the form

$$\Lambda = -\frac{1}{2} \left( \frac{\mu}{4\pi} \right)^D \int_{\mathcal{S}} \frac{d^2 \tau}{\tau_2^2} Z(\tau) \quad (5.8)$$

where  $\mathcal{S}$  denotes the semi-infinite strip in the complex  $\tau$ -plane

$$\mathcal{S} \equiv \left\{ \tau : |\operatorname{Re} \tau| \leq \frac{1}{2}, \operatorname{Im} \tau \geq 0 \right\} \quad (5.9)$$

and where the integrand  $Z(\tau)$  is

$$\begin{aligned} Z(\tau) &\equiv \tau_2^{1-D/2} \sum_n (-1)^{F_n} g_n \exp(-4\pi\tau_2 M_n^2 / \mu^2) \\ &= \tau_2^{1-D/2} \sum_n (-1)^{F_n} g_n (\bar{q}q)^{M_n^2 / \mu^2} \end{aligned} \quad (5.10)$$

with  $q \equiv e^{2\pi i \tau}$ .

As our final step, let us now demand that  $M_n = \sqrt{n} \mu$ . This assumption is valid for all of the boson/fermion configurations we have given previously, and also holds in string theory (thereby permitting us to identify  $\mu \equiv 2M_{\text{string}}$  using the standard

normalizations [14] for closed strings). Given this assumption, we can then add any term of the form

$$\tau_2^{1-D/2} \sum_{\substack{m \neq n \\ m=n \pmod{1}}} g_{mn} \bar{q}^{M_m^2/\mu^2} q^{M_n^2/\mu^2} \quad (5.11)$$

to  $Z(\tau)$  without changing the value of the cosmological constant. This is because any extra terms of the form (5.11) integrate to zero across the strip (5.9), regardless of the values of  $g_{mn}$ . We can therefore combine (5.10) and (5.11) in order to write our total integrand as

$$Z(\tau) = \tau_2^{1-D/2} \sum_{m,n} g_{mn} \bar{q}^{M_m^2/\mu^2} q^{M_n^2/\mu^2} \quad (5.12)$$

where we have defined  $g_{nn} = (-1)^{F_n} g_n$ . Note that in a supersymmetric field theory, all values of  $g_{nn}$  vanish, and hence  $Z(\tau)$  and  $\Lambda$  themselves also vanish. Also note that with or without supersymmetry,  $Z(\tau)$  has the property that  $Z(\tau) = Z(\tau + 1)$ . In other words,  $Z(\tau)$  is invariant under

$$\tau \rightarrow \tau + 1. \quad (5.13)$$

As we know, the field-theoretic cosmological constant is usually ultraviolet-divergent in the absence of supersymmetry. We have already remarked that in the Schwinger-time formalism, this ultraviolet divergence at  $p^2 \rightarrow \infty$  has been mapped to a divergence as  $t \rightarrow 0$  (or equivalently as  $\tau$  approaches the real axis in the complex  $\tau$ -plane). The fact that our strip integration region (5.9) includes a portion of this axis is the reflection of this field-theoretic divergence. The conventional method for removing this divergence in field theory without supersymmetry is to introduce an *ad hoc* ultraviolet cutoff  $\lambda$  — *i.e.*, to restrict our momentum integrations to the regions  $p^2 \leq \lambda^2$ . This cutoff procedure thereby introduces a new scale  $\lambda$  into our theory (independent of  $\mu$ ), and  $\lambda$  then serves as the high scale that sets the size of the resulting cosmological constant and likewise destabilizes the gauge hierarchy. In the language of the strip, one is essentially truncating the integration region so that  $\text{Im } \tau \equiv \tau_2 \geq \lambda$  for some  $\lambda > 0$ . In this way, the real axis is excluded.

Is there another way of truncating the integration region which essentially *avoids* introducing a new cutoff scale? Indeed, such a prescription is well known in string theory. In string theory, the coefficients  $g_{nn}$  correspond to the degeneracies of physical string states, while the off-diagonal coefficients  $g_{mn}$  correspond to the degeneracies of so-called “off-shell” or unphysical string states. Like ghosts in field theory, these states propagate only in loops and do not exist as *bona-fide* in-states or out-states. However, in string theory the physical and unphysical string states together conspire to produce values of  $g_{mn}$  in (5.12) such that  $Z(\tau)$  acquires the additional property that  $Z(\tau) = Z(-1/\tau)$ . In other words,  $Z(\tau)$  is invariant not only under (5.13), but also under the additional complex transformation

$$\tau \rightarrow -1/\tau. \quad (5.14)$$

Moreover, since the measure of integration  $d^2\tau/\tau_2^2$  in (5.8) is also invariant under the transformations (5.13) and (5.14), we see that these two transformations are a symmetry of the entire amplitude, analogous to a gauge invariance.

Of course, the symmetry generated by the transformations (5.13) and (5.14) is nothing but modular invariance, and  $Z(\tau)$  is the modular-invariant partition function of the theory. However, just as with a gauge symmetry, when calculating a scattering amplitude we must avoid overcounting by dividing out by the infinite symmetry volume factor; in other words, we must tally only those contributions which are inequivalent with respect to the symmetry. We must therefore truncate our strip region of integration so that the new (smaller) region of integration includes only one representative value of  $\tau$  up to the combined modular transformations (5.13) and (5.14). Such a region is given by

$$\mathcal{F} \equiv \left\{ \tau : |\operatorname{Re} \tau| \leq \frac{1}{2}, |\tau| \geq 1 \right\} , \quad (5.15)$$

and is commonly called the *fundamental domain of the modular group*. By excluding the real axis completely, modular invariance thus succeeds in cancelling the ultraviolet divergence in the cosmological constant without introducing a new fundamental cutoff scale beyond  $\mu$ ; essentially the modular symmetry renders the divergence spurious by enabling it to be reinterpreted as the infinite volume associated with a symmetry group. Dividing out by this volume, one thus obtains a new, manifestly finite expression for the cosmological constant:

$$\Lambda = -\frac{1}{2} \left( \frac{\mu}{4\pi} \right)^D \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau) . \quad (5.16)$$

Indeed, our discussion thus far has merely described the standard “recipe” by which one calculates the one-loop cosmological constant in string theory [15].

We have thus established that any distribution of states which causes  $Z(\tau)$  to be modular-invariant will completely cancel the ultraviolet divergence of the cosmological constant. The crux of the matter, then, is to construct such modular-invariant distributions. It is clear that the case of unbroken supersymmetry manages to do this trivially by forcing all  $g_{nn} = 0$ ; indeed, string theories with spacetime supersymmetry actually have  $g_{mn} = 0$  for *all*  $(m, n)$ , so that  $Z(\tau) = 0$  for all values of  $\tau$ . This is a level-by-level, scale-by-scale cancellation. However, our goal has been to find alternative solutions which do this in a far less trivial manner. Fortunately, we see that modular invariance does not require that  $Z(\tau) = 0$ . Thus, as long as supersymmetry is broken *in such a way that modular invariance is preserved*, the divergence in the cosmological constant can be eliminated as outlined above.

What, then, is the most general way of breaking supersymmetry while preserving modular invariance? As discussed in Sect. 4, the supersymmetry can be at most *misaligned*. Indeed, as shown in Ref. [9], the oscillating properties of misaligned supersymmetry discussed in Sect. 4 are nothing more than the residual constraints

on the degeneracies  $g_{nn}$  that follow from demanding a modular-invariant partition function  $Z(\tau)$ . These reflect themselves as a cancellation of the function forms  $\Phi(n)$ , as discussed previously.

Given these results, it might seem that one requires knowledge of both the physical ( $m = n$ ) and unphysical ( $m \neq n$ ) states in order to calculate the cosmological constant. In particular, the truncation of the region of integration from the strip  $\mathcal{S}$  to the fundamental domain  $\mathcal{F}$  in going from (5.8) to (5.16) implies that the unphysical states now make a non-zero contribution to  $\Lambda$ . However, the physical and unphysical states are related to each other via modular invariance, and it turns out that modular invariance enables the total contribution to the cosmological constant from the unphysical states to be determined directly from just the physical states. This implies that it should be possible to express our final expression for  $\Lambda$  in (5.16) directly in terms of only the diagonal elements  $g_{nn}$ , and it has been shown [16] that such an expression is given by

$$\begin{aligned}\Lambda &= -\frac{\pi}{6} \left(\frac{\mu}{4\pi}\right)^D \lim_{\tau_2 \rightarrow 0} \int_{-1/2}^{1/2} d\tau_1 Z(\tau) \\ &= -\frac{\pi}{6} \left(\frac{\mu}{4\pi}\right)^D \lim_{\tau_2 \rightarrow 0} \tau_2^{1-D/2} \sum_n g_{nn} \exp\left(-4\pi\tau_2 M_n^2/\mu^2\right) .\end{aligned}\quad (5.17)$$

This expression applies for a large class of tachyon-free string theories including all unitary non-critical strings, critical Type II strings, and heterotic strings in  $D > 2$ . Moreover, no spacetime supersymmetry is required. Since  $\Lambda$  has already been rendered finite by modular invariance, this implies that the physical-state degeneracies  $g_{nn}$  in such modular-invariant string theories must have the property that as  $\tau_2 \rightarrow 0$ ,

$$\sum_n g_{nn} \exp\left(-4\pi\tau_2 M_n^2/\mu^2\right) \sim -\frac{6}{\pi} \left(\frac{4\pi}{\mu}\right)^D \Lambda \tau_2^{D/2-1} + \dots , \quad (5.18)$$

where ‘...’ refers to terms that vanish more rapidly than  $\tau_2^{D/2-1}$  in the  $\tau_2 \rightarrow 0$  limit.

Given this result, we can now calculate the mass supertraces over the physical string states in modular-invariant theories in even numbers of spacetime dimensions [10]. In terms of the net degeneracies  $g_{nn}$ , we have

$$\begin{aligned}\text{Str } \mathcal{M}^{2\beta} &= \lim_{y \rightarrow 0} \sum_n g_{nn} (M_n)^{2\beta} e^{-yM_n^2} \\ &= \lim_{y \rightarrow 0} \left\{ \left(-\frac{d}{dy}\right)^\beta \sum_n g_{nn} e^{-yM_n^2} \right\} .\end{aligned}\quad (5.19)$$

However, in the  $y \rightarrow 0$  limit, we can use the result (5.18) to evaluate the summation over string states:

$$\text{Str } \mathcal{M}^{2\beta} = \lim_{y \rightarrow 0} \left\{ \left(-\frac{d}{dy}\right)^\beta \left[ -96\pi \frac{\Lambda}{\mu^2} (4\pi y)^{D/2-1} + \dots \right] \right\} . \quad (5.20)$$

Consequently, we see that  $\text{Str } \mathcal{M}^{2\beta}$  must necessarily *vanish* for all  $0 \leq \beta < D/2 - 1$ :

$$\text{Str } \mathcal{M}^0 = \text{Str } \mathcal{M}^2 = \dots = \text{Str } \mathcal{M}^{D-4} = 0 . \quad (5.21)$$

Moreover, for even spacetime dimensions, we see that the first non-zero supertrace is given by

$$\text{Str } \mathcal{M}^{D-2} = 24 (-4\pi)^{D/2} (D/2 - 1)! \frac{\Lambda}{\mu^2} . \quad (5.22)$$

Thus, the value of the first non-zero supertrace is set by the one-loop cosmological constant!

For  $D = 10$ , these results imply that  $\text{Str } \mathcal{M}^0$ ,  $\text{Str } \mathcal{M}^2$ ,  $\text{Str } \mathcal{M}^4$ , and  $\text{Str } \mathcal{M}^6$  all vanish, even if supersymmetry is not present. This is the origin of the result (4.2) claimed earlier. On the other hand, for  $D = 4$ , these relations reduce to

$$\text{Str } \mathcal{M}^0 = 0 , \quad \text{Str } \mathcal{M}^2 = 24 (4\pi)^2 \frac{\Lambda}{\mu^2} . \quad (5.23)$$

In each case, these relations are *guaranteed* for any modular-invariant theory regardless of its method of construction, its compactification manifold, or its low-energy phenomenology. No supersymmetry is required, softly broken or otherwise.

As discussed in Ref. [10], these results may seem extremely counter-intuitive from the point of view of ordinary four-dimensional field theory. After all, these results imply that we can relate the actual value of the one-loop amplitude  $\Lambda$  to a single (mass)<sup>2</sup> supertrace. By contrast, in ordinary quantum field theory, a supertrace of this order should describe the *quadratic divergence* of such an amplitude, not its constant term. Indeed, in ordinary quantum field theory, we know that  $\text{Str } \mathcal{M}^0$ ,  $\text{Str } \mathcal{M}^2$ , and  $\text{Str } \mathcal{M}^4$  respectively govern quartic, quadratic, and logarithmic divergences in  $\Lambda$ . The point here, however, is that modular invariance is so powerful a symmetry that it effectively softens the divergences of amplitudes by four powers of mass. Thus, in a theory with modular invariance, the quartic and quadratic divergences are automatically cancelled — even without supersymmetry. Moreover, the highest remaining divergence for such an amplitude is a logarithmic one, now governed by  $\text{Str } \mathcal{M}^0$ . The vanishing  $\text{Str } \mathcal{M}^0$  in these scenarios thus guarantees the finiteness of  $\Lambda$ , and  $\text{Str } \mathcal{M}^2$  then describes the constant term — *i.e.*, the value of  $\Lambda$  itself. Moreover, as a byproduct, these results also verify that the regulator chosen for our supertraces in (3.5) respects modular invariance, as originally claimed.

For our purposes, however, the most important observation is that *the gauge hierarchy problem is now related to the cosmological constant problem at a fundamental level*. Indeed, the only way to ensure that both  $\text{Str } \mathcal{M}^0$  and  $\text{Str } \mathcal{M}^2$  vanish is to ensure that  $\Lambda = 0$ . Thus, at least at the one-loop level, *modular invariance and misaligned supersymmetry tie these two fundamental problems together* in precisely the way we envisioned in Sect. 2.



Moreover, in string theory, a non-zero one-loop cosmological constant signifies an unstable string vacuum. This occurs because a non-zero one-loop zero-point function (cosmological constant) necessarily implies a non-zero one-loop dilaton tadpole. Thus, in string theory, the cosmological constant and gauge hierarchy problems each become tantamount to the third problem of vacuum stability! In other words, they are related to the moduli problem. Of course, the relation of the cosmological constant problem to the string moduli problem is not new. Our point here, however, is that in string theory, the problems of vacuum stability, cosmological constant, and gauge hierarchy are now seen to be merely *one* problem, not three separate problems. Thus, any stable non-supersymmetric string will *automatically* incorporate solutions to the remaining problems as well.

We have demonstrated these claims merely to one-loop order for strings in a flat background spacetime. However, it is likely that multi-loop generalizations of modular symmetry will guarantee the continuation of these connections to all orders, and perhaps even non-perturbatively. Indeed, just like misaligned supersymmetry itself, it is likely that such the connection between stability, the cosmological constant, and the gauge hierarchy supertraces is part of the consistency of non-supersymmetric strings.

Needless to say, all of these results hold in a trivial manner even when an exact supersymmetry is present. In the presence of unbroken spacetime supersymmetry, the string vacuum is automatically stable (living along a flat potential in all directions), and the technical gauge hierarchy and cosmological constant problems are automatically solved as a result of this flatness. Our claim, then, is that a similar situation persists even when the supersymmetry is broken: as long as a stable non-supersymmetric string vacuum exists, that stability will manifest itself through the appearance of a misaligned supersymmetry, and this misaligned supersymmetry will be sufficient to guarantee a solution to the technical gauge hierarchy and cosmological constant problems.

Given these observations, the next step is clearly to find a stable non-supersymmetric string, or equivalently to find a string theory with  $\Lambda = 0$ . Moreover, for our purposes, we wish to demand that such a string not even exhibit boson/fermion degeneracies (so that  $g_{nn} \neq 0$ ). While the existence of such a string theory is not known at present, we wish to offer the following observation. In Sect. 4, we introduced two four-dimensional non-supersymmetric tachyon-free heterotic string models, one with gauge group  $SU_6 \times (SU_4)^3 \times (SU_2)^9 \times U_1$  and the other with gauge group  $E_6 \times SO_{10} \times (SU_4)^3 \times (U_1)^2$ . Even though these string models have unequal gauge groups, unequal particle contents, and unequal bosonic and fermionic degeneracies at all mass levels, it turns out that they manage to have exactly the same one-loop cosmological constant! In other words, their one-loop cosmological constants are exactly equal,

$$\Lambda_{SU_6 \times (SU_4)^3} = \Lambda_{E_6 \times SO_{10}} , \quad (5.24)$$

even though their partition functions are unequal,

$$Z_{SU_6 \times (SU_4)^3} \neq Z_{E_6 \times SO_{10}} . \quad (5.25)$$

This sort of non-trivial degeneracy between non-supersymmetric string models was originally noticed in Ref. [17], and occurs quite frequently in the moduli space of non-supersymmetric string theories. Given this, it is possible to consider the *difference* of partition functions

$$\Delta Z \equiv \frac{1}{4} \left[ Z_{SU_6 \times (SU_4)^3} - Z_{E_6 \times SO_{10}} \right] \quad (5.26)$$

in order to generate solutions  $\{g_{nn}\}$  which have the property that  $\Lambda = 0$ . Specifically, given the partition functions of these string models, we find

$$\begin{aligned} \Delta Z = & \frac{1}{128 \tau_2} \frac{1}{\bar{\eta}^{12} \eta^{24}} \times \\ & \times \sum_{\substack{i,j,k=2 \\ i \neq j \neq k}}^4 |\vartheta_i|^4 \left\{ \vartheta_i^4 \vartheta_j^4 \vartheta_k^4 \left[ 2 |\vartheta_j \vartheta_k|^8 - \vartheta_j^8 \overline{\vartheta_k}^8 - \overline{\vartheta_j}^8 \vartheta_k^8 \right] \right. \\ & \left. + \vartheta_i^{12} \left[ 4 \vartheta_i^8 \overline{\vartheta_j}^4 \overline{\vartheta_k}^4 + (-1)^i 13 |\vartheta_j \vartheta_k|^8 \right] \right\} \end{aligned} \quad (5.27)$$

where  $\eta$  and  $\vartheta_i$  are the Dedekind eta-function and Jacobi theta-functions:

$$\begin{aligned} \eta(q) &\equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) = \sum_{n=-\infty}^{\infty} (-1)^n q^{3(n-1/6)^2/2} \\ \vartheta_2(q) &\equiv 2q^{1/8} \prod_{n=1}^{\infty} (1 + q^n)^2 (1 - q^n) = 2 \sum_{n=0}^{\infty} q^{(n+1/2)^2/2} \\ \vartheta_3(q) &\equiv \prod_{n=1}^{\infty} (1 + q^{n-1/2})^2 (1 - q^n) = 1 + 2 \sum_{n=1}^{\infty} q^{n^2/2} \\ \vartheta_4(q) &\equiv \prod_{n=1}^{\infty} (1 - q^{n-1/2})^2 (1 - q^n) = 1 + 2 \sum_{n=1}^{\infty} (-1)^n q^{n^2/2} . \end{aligned} \quad (5.28)$$

We can then do a power expansion of  $\Delta Z$ ,

$$\Delta Z(q, \bar{q}) = \frac{1}{\tau_2} \sum_{m,n} g_{mn} \bar{q}^m q^n , \quad (5.29)$$

and thereby generate a set of physical-state degeneracies  $\{g_{nn}\}$  for which the corresponding cosmological constant vanishes identically:

$$\Lambda = \int_{\mathcal{F}} \frac{d^2 \tau}{\tau_2^2} \Delta Z(\tau, \bar{\tau}) = 0 . \quad (5.30)$$

Moreover, upon rescaling  $n \rightarrow 4n$ , we find that these degeneracies  $\{g_{nn}\}$  are precisely those of the “magic” solution presented in Sect. 3, for which both  $\text{Str } \mathcal{M}^0$  and  $\text{Str } \mathcal{M}^2$  vanish identically!

Of course, the difference  $\Delta Z$  of two string partition functions is not necessarily another string partition function. In other words, the set of integers  $\{g_{nn}\}$  need not necessarily (and in this case does not) emerge as the state degeneracies of a stable non-supersymmetric string. Despite this fact, this solution for  $\{g_{nn}\}$  continues to have vanishing supertraces and corresponds to a vanishing cosmological constant in the manner described above. This solution is therefore perfectly valid from a purely field-theoretic point of view.

For this reason, it is instructive to ascertain how these  $\{g_{nn}\}$  manage to achieve a vanishing cosmological constant. It turns out that the underlying mechanism rests on the fact that the function  $\Delta Z$  in (5.27) exhibits a so-called Atkin-Lehner symmetry [18]. Specifically, for such a function  $\Delta Z$ , it is possible to rewrite the integral in (5.30) in such a way that the new integrand is odd under a discrete *non*-modular transformation such as  $\tau \rightarrow -1/2\tau$  while the new integration measure and domain are even under this transformation. The resulting integral thus vanishes as a result of a discrete selection rule. Indeed, such Atkin-Lehner symmetries are possible only in theories with infinite numbers of states, and once again such cancellations emerge as the result of delicate conspiracies between physics at all mass scales. Unfortunately, several attempts [19] and ultimately a no-go theorem [20] have shown that no self-consistent string model can have a partition function exhibiting such an Atkin-Lehner symmetry. While a proposal has been made [21] for generalizing the idea of Atkin-Lehner symmetry to evade this no-go theorem, no stable self-consistent non-supersymmetric string model has yet been constructed along these lines.

More recently, it has been shown [22] that there exist non-supersymmetric compactifications of Type II strings for which  $\Lambda = 0$  to one- and two-loop orders. Moreover, it has been conjectured [22, 23] that this cancellation persists to higher loops as well, and perhaps even non-perturbatively. As such, this is an exciting development. However, even though these models are non-supersymmetric, they nevertheless exhibit an exact level-by-level boson/fermion degeneracy. They therefore have  $g_{nn} = 0$  for all  $n$ . Thus, even though they are non-supersymmetric, they are not satisfactory for our phenomenological purpose of being able to accommodate, for example, the Standard Model among their low-energy states.

Thus, the critical issue of whether there exist stable non-supersymmetric strings *without* boson/fermion degeneracies remains unknown. However, our point is that if such a string is found, it will necessarily exhibit a misaligned supersymmetry as discussed above. It will therefore already incorporate solutions to the technical gauge hierarchy and cosmological constant problems simultaneously, in the sense described above. If nothing else, this observation adds urgency to the search for a stable, non-supersymmetric string.

## 6 Discussion and open questions

In this paper, we have proposed an alternative perspective concerning the gauge hierarchy and cosmological constant problems. Rather than address these problems through the language of a low-energy effective field theory comprising a finite number of states, we have proposed an alternative solution in which an infinite number of states at all energy scales conspires to remove the quantum-mechanical sensitivity to high scales that would otherwise appear to exist in the calculations of the Higgs mass and the cosmological constant. As discussed above, the critical ingredient in this approach is modular invariance and the misaligned supersymmetry that ensues in the spectrum of physical string states. As such, this proposal would be realized naturally within the context of a stable, non-supersymmetric string. Indeed, one of the advantages of this approach is its generality: since our observations are built only on modular invariance and on the ensuing misaligned supersymmetry, they apply to all (stable) non-supersymmetric closed strings regardless of their method of construction or other phenomenological properties.

### 6.1 Alternative approach to string phenomenology?

Needless to say, assuming that a such a string exists, the proposals in this paper favor a corresponding alternative approach to string phenomenology. As sketched in Fig. 3 [paths (a) and (b)], the traditional approach to string phenomenology [24] has always been to begin with a supersymmetric string model at the Planck scale, and then essentially to integrate out the heavy string states, leaving behind a supersymmetric effective field theory (*e.g.*, the Minimal Supersymmetric Standard Model) comprising only the light (or massless) string degrees of freedom. As we have seen, this process of integrating out the heavy string states eliminates the fundamental finiteness properties that are intrinsic to the full string theory as a result of a conspiracy between the states at all energy scales. Then, as a second step, one typically breaks supersymmetry in this effective field theory through some field-theoretic mechanism (*e.g.*, gaugino condensation), ultimately resulting in the non-supersymmetric Standard Model. It is this final theory which has an apparent gauge hierarchy problem.

By contrast, the ideas in this paper favor an alternative approach [indicated along paths (c) and (d)] in which supersymmetry is broken first [25, 26]. Unlike path (a), such supersymmetry breaking occurs directly within the context of the *full* string theory through a mechanism (such as Scherk-Schwarz compactification [25]) which respects the full underlying string symmetries, including worldsheet modular invariance.\* Assuming that such a stable non-supersymmetric string exists, this procedure

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\* In this connection, we emphasize that we are discussing *worldsheet* modular invariance. In the literature one often encounters discussions of modular-invariant supergravities and modular-invariant gaugino condensation (see, *e.g.*, Ref. [27]), but those discussions involve modular  $SL(2, \mathbb{Z})$

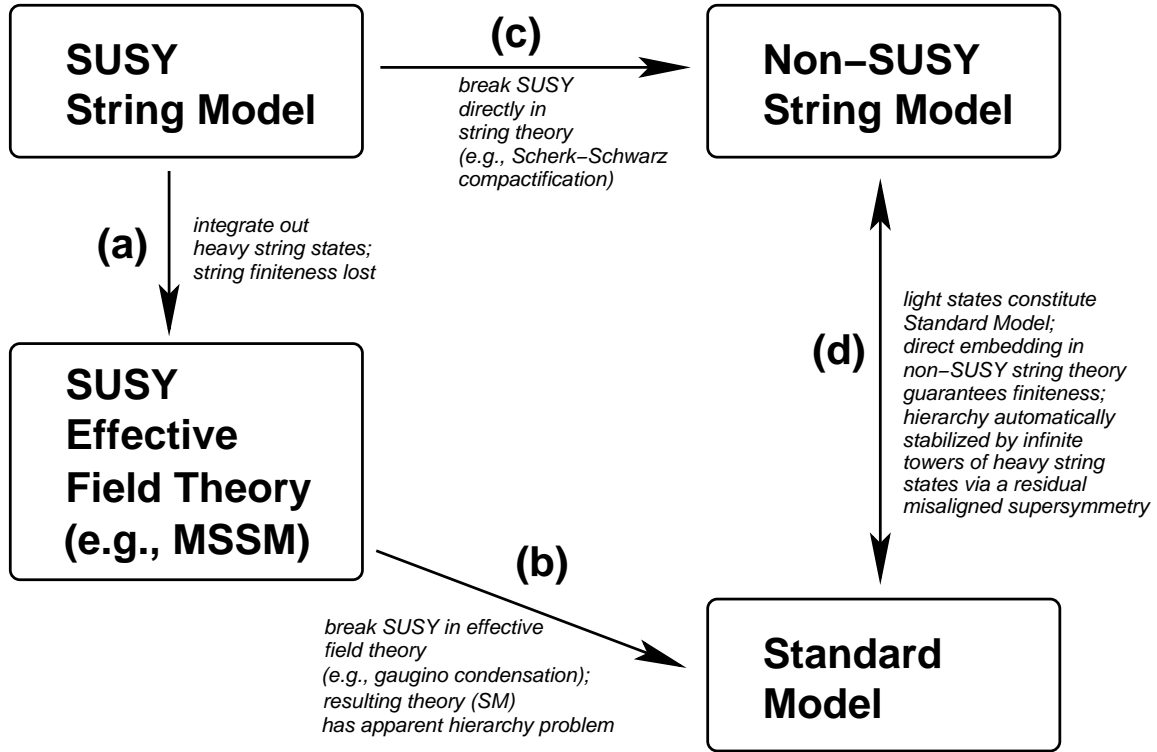


Figure 3: Two approaches to string phenomenology. In the traditional approach [paths (a) and (b)], the Standard Model is realized only after supersymmetry is broken in an effective theory (*e.g.*, the MSSM) which is itself derived as the low-energy limit of a supersymmetric string. In the alternative approach [paths (c) and (d)], supersymmetry is broken first in a manner that preserves the full string symmetries that underlie string finiteness. The light degrees of such a string theory would then constitute the Standard Model directly, and the gauge hierarchy would automatically be stabilized through a misaligned supersymmetry.

ensures that the original finiteness properties of the string theory remain intact. In such a scenario, the light states of this string would then constitute the Standard Model directly, and the direct embedding of the Standard Model within such a non-supersymmetric string would automatically stabilize the gauge hierarchy in the manner we have been discussing.

In principle, both of these approaches provide a connection between a string theory at the Planck scale and the non-supersymmetric Standard Model at lower scales. However, the physics *beyond* the Standard Model clearly depends on the route taken. The traditional route [paths (a) and (b)] requires low-energy (weak-scale) supersymmetry in order to protect the gauge hierarchy, and does not directly address symmetries of fields in the *target space*.

the cosmological constant problem. By contrast, the alternative route [paths (c) and (d)] does not require supersymmetry at any energy scale either at or below the string scale. The gauge hierarchy and cosmological constant problems are instead directly related to the conjectured stability of the non-supersymmetric string via the misaligned supersymmetry that remains in the full string spectrum.

Of course, one interesting possibility may be that the diagram in Fig. 3 actually *commutes*. In other words, it is possible that after gaugino condensation [path (b)], the states that emerge are still realizable as the low-energy states of a non-supersymmetric string in which the scale of supersymmetry breaking is somehow much smaller than the string scale.<sup>†</sup> However, this would require that the *field-theoretic* mechanism of gaugino condensation somehow preserve the full *string-theoretic* symmetries (such as worldsheet modular invariance) that underlie string finiteness. Whether this is possible remains unclear.

## 6.2 Relevance for the “brane world”?

Our approach may also be relevant for the so-called “brane world” scenario of physics beyond the Standard Model [3, 4, 5]. At first glance, our approach is a “no-braner”, since the Standard Model is not necessarily restricted to any particular subspace of the compactified string theory. However, our approach may be useful even within brane-world scenarios with reduced GUT, Planck, and string scales. Since our results apply for closed strings, they are applicable in the “bulk” of extra large dimensions, where all degrees of freedom are necessarily realized as excitations of closed strings. As such, it might be possible to follow this approach (rather than supersymmetry) in order to stabilize bulk dynamics and ensure finiteness in the bulk. However, it is possible that generalizations of our results would also apply in the open-string sectors of Type I string models. If so, then this approach could be used to ensure finiteness on the brane as well as in the bulk, and permit the fundamental string scale to exceed the TeV scale in such scenarios. In this context, it would be interesting to study the relation between this approach and the phenomenon of brane supersymmetry-breaking [28].

Of course, the precise low-energy phenomenology of our scenario depends on the value of the string scale  $\mu$ . *A priori*, this scale is a free parameter in our approach, since the misaligned supersymmetry and the cancellation of the mass supertraces holds regardless of the value of the fundamental scale  $\mu$ . If  $\mu$  is large (*e.g.*, near the usual Planck scale), then at low energies we would not expect to detect new physics other than the “extra” light states (beyond the Standard Model) that generically emerge in quasi-realistic string models. On the other hand, if  $\mu$  is near the TeV range, then the full spectrum of string states would become accessible to upcoming collider experiments, and the appearance of a misaligned supersymmetry should become experimentally verifiable. Of course,  $\mu$  may also take intermediate values [29].

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<sup>†</sup>Models of this sort can be found in Ref. [2].

### 6.3 Open issues

Needless to say, this approach raises a number of outstanding issues. As such, the comments that follow are highly speculative.

First, as mentioned above, our specific results are proven merely to one-loop order for closed strings in a flat background spacetime. In other words, misaligned supersymmetry and the corresponding supertrace relations such as those in (5.21), (5.22), and (5.23) hold for tree-level masses and one-loop cosmological constants. It is still necessary to demonstrate that misaligned supersymmetry and the vanishing of the cosmological constant persist to all orders, and perhaps even non-perturbatively. Of course, as discussed in Sect. 5, this is nothing but the usual moduli problem. However, the root of our approach is ultimately modular invariance, and this is a symmetry with well-understood multi-loop generalizations. It is therefore likely that multi-loop generalizations of modular symmetry will continue to tie the hierarchy and cosmological constant problems together, and relate them ultimately to the overriding (unsolved) problem of non-supersymmetric vacuum stability.

A second, closely related issue concerns the extension of these results to open strings. Although modular invariance can no longer be expected to hold, open strings can be realized as orientifolds of closed strings [30]. As such, remnants of misaligned supersymmetry may survive the orientifolding procedure. Indeed, one of the critical ingredients in our supertrace derivation is the result (5.17), which rests directly on modular invariance. However, this result has recently been generalized to open strings [31].

Moreover, in many instances open strings emerge as the strong-coupling duals of closed strings [32], and such strong/weak coupling duality relations have been conjectured to hold even without supersymmetry [33]. In such instances, a successful generalization of our results to open strings would therefore allow us to conclude that that our closed-string results would hold even non-perturbatively.

A third open issue concerns the origin of the scale of electroweak symmetry breaking. Clearly, our approach does not shed light on this critical issue. This scale is presumably set by some dynamics connected with the stability of the underlying non-supersymmetric string. However, regardless of how this scale is set, our point is that it is then guaranteed to be insensitive to other heavy scales such as the string scale. Thus, once this scale is set, the *technical* hierarchy problem is solved, and any apparent quantum-mechanical sensitivity to heavy scales is merely an artifact of having truncated our theory to include only the light degrees of freedom.

Similar comments may also apply to the cosmological constant problem. Of course, our results assume the propagation of our strings on a flat background spacetime; in such cases the one-loop condition for vacuum stability becomes the vanishing of the one-loop cosmological constant. In some sense, this may be considered to be an automatic “self-tuning” since all other non-supersymmetric strings are necessarily unstable (with non-zero dilaton tadpoles) and thus necessarily flow in string mod-

uli space until they reach a stable point. Of course, how this generalizes to higher loops and curved spaces is more difficult to study (especially since string theory undoubtedly demands more than a semi-classical treatment of the background fields). However, it is likely that a true solution to the cosmological constant problem will ultimately require physics that is intrinsically *stringy*, and which therefore cannot be captured within a semi-classical treatment of background gravitational fields (alone or in the presence of branes).

Nevertheless, one interesting observation concerns phase transitions in the early universe. As discussed above, our assumption in this paper has been that our present-day low-energy world can be directly realized as the low-energy states of a non-supersymmetric string. If this assumption holds at each epoch in the evolution of the universe, then all phase transitions should occur in such a way that they do not violate fundamental string symmetries such as worldsheet modular invariance. In other words, unless modular invariance is broken spontaneously, modular invariance should continue to protect the cancellations we have observed by causing the infinite towers of string states to experience phase transitions and adjust their properties whenever the low-energy states do the same. Thus, the “self-tuning” described above would be maintained, and modular invariance would be preserved *through* all phase transitions, including the QCD phase transition [34].

A fourth outstanding issue concerns the relevance of effective field theory, particularly as it relates to the phenomenological study of theories (such as string theories) in which there are infinite numbers of heavy states. Needless to say, our approach to the gauge hierarchy problem is one which goes against the spirit of effective field theory. Unlike other approaches to this problem (such as supersymmetry), our approach rests on conspiracies between physics at all energy scales simultaneously, and illustrates the apparent fallacy of integrating out *infinite* numbers of heavy string states. Or, phrased somewhat differently, the low-energy effective field theory derived from the string is one in which certain parameters (*e.g.*, supertraces) are magically cancelled by physics that cannot be captured within an approach based purely on low-energy physics. It would therefore be interesting to clarify for which classes of phenomenological problems in string theory an effective field theory approach is valid, and for which classes of problems a treatment within the framework of the full string theory is required. Clearly problems involving relations between widely separated scales (such as the gauge hierarchy and cosmological constant problems) are likely to be in the second category.

In this connection, note that the cosmological constant problem is usually considered to be an infrared problem. By contrast, in string theory, there is no distinction between ultraviolet and infrared physics, since modular invariance exchanges the two and provides a direct connection between low-energy states and those which are infinitely heavy. It is therefore natural that our approach to the cosmological constant problem is one which necessarily involves all energy scales simultaneously. Similar remarks may also apply to solutions to the cosmological constant problem which are



based on the holographic principle [35].

Conversely, it may also be interesting to study the extent to which modular invariance may be exploited as a new regularization mechanism entirely within a field-theoretic context (albeit a field theory with an infinite number of states). In some sense, one may think of modular invariance as providing an infinite-component Pauli-Villars regulator in which infinite numbers of heavy fields cancel the divergences of light fields. It would be interesting to explore the connection between this method of achieving finiteness and finiteness achieved via soft supersymmetry breaking [36]. There may also be interesting connections to other non-traditional field-theoretic regularization mechanisms such as “non-local regularization” [37] and “Kaluza-Klein regularization” [38]. Indeed, misaligned supersymmetry may specifically be able to shed light on some of the unsolved features involved in the latter.

The ideas we have proposed in this paper are clearly speculative. At the very least, they rest critically on the existence of stable non-supersymmetric strings, a fact which has not yet been demonstrated. However, our main point is that any such string theory must necessarily exhibit the properties we have discussed, since modular invariance and misaligned supersymmetry are intrinsic ingredients in the self-consistency of string theory. As such, *this* appears to be the path to finiteness chosen by string theory. For this reason alone, we believe that this approach merits further exploration.

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